Univ. Coll
Toronto.
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EXAMPLES

OF

THE APPLICATION OF TRIGONOMETRY

TO

Crystallographic Calculations.

Drawn up for the use of Students in the University of Toronto,

By E. J. CHAPM ,

PROFESSOR OF MINERALOGY AND GEOLOGY IN UNIVERSITY COLLEGE:

(Late Professor in University College, London.)

TORONTO:

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The Application of Trigonometry to Crystallographic Calculations having been adopted by the University of Toronto, as one of the "Honor subjects" in the Department of Mineralogy and Geology, for students of the Fourth Year,* the accompanying Examples have been drawn up to convey a general idea of the principles involved in this application: more especially with a view to assist candidates who may not be able to attend the regular lectures given on the subject in University College.

The Trimetric System has been selected, with regard to these Examples, as the best adapted to exhibit the nature of Crystallographic Calculations in general. *A knowledge of the common principles of Crystallography, and of Plane and Spherical Trigonometry—so far, at least, as regards the working out of ordinary cases—is, of course, pre-supposed on the part of the student.* As many persons, however, may find it desirable to take up this branch of inquiry, without being much versed in mathematical investigations, a few explanatory hints have been added here and there, and the calculations have been rendered as simple as the nature of the subject will admit. Abbreviated formulæ have been thus avoided; and the calculations are elucidated, moreover, by various original diagrams, designed expressly for this memoir. If these examples be thoroughly mastered, the solution of kindred questions in other systems of crystallization will occasion no difficulty.

University College,
Toronto, March, 1860.

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* The additional honor-subject in this Department, for University Students of the Fourth Year, comprises the Geology of North America, with the Rock Formations, Fossils, and Economic Minerals of Canada considered in detail. The subject of Crystallography, it should be observed, has not only important bearings on Mineralogy proper, but on Chemistry also, as well as on Optics, and other branches of Physical Science.

☞ The writer's best thanks are due to Messrs. Lovell and Gibson, for the care with which the symbols of the various crystal-forms described in the following pages, have been printed. The setting up of these symbols, with ordinary type, is a work involving much skill and labour.

EXAMPLES

OF THE

Crystallographic Applications

OF

TRIGONOMETRY.

TRIMETRIC SYSTEM OF CRYSTALLIZATION.

1. In this System—the *Rhombic System* of Naumann, *Prismatic System* of Miller—the three axes are of unequal lengths. They cross one another at right angles in the centre of the crystal. As shewn in figure 1, the vertical axis may be conveniently denoted by the symbol x; the longer horizontal (the macrodiagonal, or right and left axis) by the symbol $\bar{x}$; and the shorter horizontal (the brachydiagonal, or back and front axis) by the symbol $\breve{x}$.

2. The forms of this system (see § 3,) comprise *Basal, Polar,* and *Vertical* Forms. The Basal form (B) consists of but two opposite planes, placed horizontally, one at each extremity of axis x. The Polar, or Pyramidal forms, are of three general kinds :—*Front polars* ($= m\bar{P}$) or "Macrodomes," between x and $\breve{x}$, and consequently parallel to $\bar{x}$; *Side polars* ($= m\breve{P}$) or "Brachydomes," between x and $\bar{x}$, and consequently parallel to $\breve{x}$; and intermediate polars, or *polars* properly so-called (Octahedrons, &c.) between x, $\bar{x}$, and $\breve{x}$. These latter are of two kinds, $=$ m P, m P n : they cut all the axes. The vertical forms are also of three general kinds: *front verticals* ($= \bar{V}$) comprising merely two tangent planes,* one at each extremity of

* In Crystallography, a plane is said to be tangent to an axis, when the plane is placed rectangularly at the point of the axis referred to. In the Trimetric system, there are three pairs of these tangent planes:—B, placed horizontally at the ends of axis x; and $\bar{V}$ and $\breve{V}$ placed vertically at the extremities of $\breve{x}$ and $\bar{x}$, respectively.

axis $\breve{x}$, and consequently parallel to x and $\bar{x}$; *side verticals* ($=\breve{V}$) also of two planes, tangents at the extremities of axis $\bar{x}$, and consequently parallel to x and $\breve{x}$; and intermediate verticals, or *verticals* properly so-called, between $\bar{x}$ and $\breve{x}$, and consequently parallel to x. These latter are denoted by the symbols V, and Vn.

In all of the above symbols, m stands for the length of the vertical axis x, and n for the length of the shorter horizontal axis $\breve{x}$, compared with that of the longer horizontal axis $\bar{x}$ as unity.

3. The crystals of this system may be reduced to four simple types: *Right Rectangular Prisms,* made up of the basal planes and front and side verticals (fig. 2); *Right Rhombic Prisms,* made up of the basal and vertical planes (fig. 3); *Rectangular octahedrons* (base a rectangle), made up of the front and side polars (fig. 4); and *Rhombic Octahedrons* (base a rhomb), made up of the polar planes properly so-called, or those which cut the three axes (fig. 5).

The relations of these crystals to one another, are shewn by the lettering on the figures.*

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* The correspondence of the notation employed in our figures, with the symbols adopted by Naumann and Dana, is shewn in the following table:

Chapman:	*Naumann:*	*Dana:*
B =	O P	O
$\bar{V}$ =	$\infty\bar{P}\infty$	$i\bar{i}$
$\breve{V}$ =	$\infty\breve{P}\infty$	$i\breve{i}$
V =	$P\infty$	I
$\bar{P}$ =	$\bar{P}\infty$	$\bar{1}$
$\breve{P}$ =	$\breve{P}\infty$	$\breve{1}$
P =	P	1

The accompanying Table may also prove useful to the student. The sign ∞, the usual sign of *infinity,* when attached to the symbol of an axis, denotes the form to be parallel to that axis, or to cut it at the distance infinity:

B	=	$\bar{\infty}\ \breve{\infty}$	=	$x,\ \infty\bar{x},\ \infty\breve{x}$
$\bar{V}$	=	$\infty\ \bar{\infty}$	=	$\infty x,\ \infty\bar{x},\ \breve{x}$
$\breve{V}$	=	$\infty\ \breve{\infty}$	=	$\infty x,\ \bar{x},\ \infty\breve{x}$
V	=	∞	=	$\infty x,\ \bar{x},\ \breve{x}$
$\bar{P}$	=	$\bar{\infty}$	=	$x,\ \infty\bar{x},\ \breve{x}$
$\breve{P}$	=	$\breve{\infty}$	=	$x,\ \bar{x},\ \infty\breve{x}$
P	=	$\cdot$	=	$x,\ \bar{x},\ \breve{x}$

The use of the symbols B, V, &c., has this advantage: it enables us to transform the signs into words with great readiness—the signs and the names of the forms which they represent being more or less alike. Thus B=*B*, or *Base;* $\bar{V}$=*Front V*, or *Front Vertical;* $\breve{V}$=*Side V*, or *Side Vertical;* and so on with regard to the rest.

4. The map-diagram, figure 6, exhibits the relative positions of all the forms belonging to this system. The observer is supposed to be looking down at the top of the crystal, parallel to the common plane of the two horizontal axes, $\bar{x}$ and $\breve{x}$. The vertical or prismatic forms—those parallel to the vertical axis—lie on the outside of the ellipse. Owing to want of space, the signs , ,, ,,, have been substituted, respectively, for $\frac{1}{m}$ P*n*, P*n* and *m* P*n*; and for $\frac{1}{m}$ P $\frac{1}{n}$, P $\frac{1}{n}$, *m* P $\frac{1}{n}$. Figure 6*a*, on the next plate, is a vertical projection, shewing the same forms. A portion only of the projection is given, the other parts (as will be seen by an inspection of the horizontal projection, fig. 6,) being merely repetitions of this.

5. In calculating the axial ratios, &c., of crystals, by means of trigonometry, we have in every case to subdivide the crystal (or that portion of it to which the calculations refer,) into one or more plane or spherical triangles—from the known angles and sides of which, the other angles and sides may be deduced.

6. Figure 7 represents a crystal of pyrolusite, containing the basal form, B, and two vertical prisms. The axial relations of these latter have to be determined. The inclination of the two outer faces (V) in front = 93° 40°. Half this equals the angle *A* in fig. 8.* As the axes cross at right angles, *C*, in this figure, = 90°. Axis $\bar{x}$ is considered equal to unity. Hence, to determine axis $\breve{x}$ (or the side *b*,) we have the formula :

$$R : \cot A :: a : b$$

$$\therefore \text{Log } b = (\log \cot 46^\circ\ 50') - 10 = \bar{1}.9721882 = \log 0.9380.$$

The inclination of the two inner faces over a front edge = 129° 46′. Half this, as before, = angle *A′* in fig. 8. And, consequently,

$$\text{Log } b = (\log \cot 64^\circ\ 53') - 10 = \bar{1}.6709774 = \log 0.4688.$$

In one of these forms, therefore, the shorter axis compared to the longer axis as unity, is just twice as great as in the other. As the outer form is by far the more common of the two, we assume it to be the protaxial prism, and give it the simple symbol V. The inner form then becomes V$\frac{1}{2}$. See the remarks at the close of § 7.

7. In figure 9, a crystal of sulphate of lead is represented, com-

* In this figure the side *a* represents half $\bar{x}$; and the side *b*, half $\breve{x}$. The former equals 1 or unity, and the latter has to be determined. The *semi-axes* employed in these constructions, bear, of course, to each other, the same relations as the entire axes.

prising the base, a vertical prism, two front polars or macrodomes, and a side polar or brachydome.

$V : V = 103° 38'$.
$B : \check{P} = 127° 45'$.
$B : \frac{1}{4}\bar{P} = 157° 33'$.
$B : \frac{1}{2}\bar{P} = 140° 27'$.

To determine axis $\check{x}$ ($\bar{x}$ being unity,) we have $\frac{V : V}{2} = 51° 49'$. Then— (See Fig. 8.)

(Log cot 51° 49′) − 10 = $\bar{1}$.8956719 = log 0.78645 = log $\check{x}$.

To determine axis x, we assume the side polar $\check{P}$ to be a protaxial form: this side polar being of almost constant occurrence, and often predominating in the crystals. B : $\check{p}$ = 127° 45′. This, less 90° = the angle A in the diagram, fig. 10. Consequently (axis $\bar{x}$ being unity):

R : cot 37° 45′ :: 1 : x : whence:

Log x = (log cot 37° 45′) − 10 = 0.113[illegible]22 = log 1.29[illegible]. (50)

Turning now to the two front polars, we find the inclination of the base on the one adjacent to it = 157° 33′. Deducting 90° from this, we get the angle A' in fig. 11. Then, to obtain the vertical axis x, we have the formula:

R : cot A' :: $\check{x}$: x.

Log $\check{x}$ (as already found,)	=	$\bar{1}$.8956719
Log cot 67° 33′ - -	=	9.6161514
		9.5118233
Log R - - - - - -	=	10
		$\bar{1}$.5118233 = log 0.3250.

This value being just one fourth that of x in the protaxial form, the symbol of this front polar, or macrodome, becomes $\frac{1}{4}\bar{P}$.

The inclination of the base on the lower form = 140° 27′. Deducting 90° from this, and proceeding as before, we obtain:

Log $\check{x}$ - - -	=	$\bar{1}$.8956719
Log cot. 50° 27′	=	9.9168765
		9.8125484
Log R - - -	=	10
		$\bar{1}$.8125484 = log 0.6418

This value being half that of x in the protaxial form, the symbol becomes $\frac{1}{2}\bar{P}$.

Note.—The assumption of this protaxial form or ***starting-point***, is, of course more or less arbitrary. As a general rule, we select a form of common occurrence, or one that predominates in the combinations; or otherwise, one to which the cleavage planes are parallel. Any one form, however, being chosen, the axial ratios of all the other forms belonging to the substance, will bear some simple relation to it. Thus, if the lower front polar were assumed to bear the symbol $\bar{P}$, the upper front polar (in which $x = .3250$) would be $\frac{1}{2}\bar{P}$; and the side polar (with $x = 1.299$) would be $2\breve{P}$. In like manner, if the upper front polar were taken as a starting point ($= \bar{P}$, *id est*, $1\bar{P}$), the lower front polar would be $2\bar{P}$; and the side polar, $4\breve{P}$.

8. Fig. 12 represents a crystal of sulphur: a combination of three rhombic octahedrons or polars, each face cutting the three axes. The measured inclinations are as follows:

P on P	Over front edge...	106° 38′.
	Over side edge ...	84° 58′.
	Over middle edge	143° 17′.
$\frac{1}{3}$ P : $\frac{1}{3}$ P	Over front edge	127°
	Over side edge ...	113° 10′.
	Over middle edge	90° 16′.
$\frac{1}{5}$ P : $\frac{1}{5}$ P	Over front edge...	142° 4′.
	Over side edge ...	132° 44′.
	Over middle edge	62° 9′.

To calculate the axial ratios of these forms, we construct the spherical triangle, figure 13, in which $A =$ half the inclination over a front edge; $B =$ half the inclination over a side edge; and $C = 90°$, or the meeting of two sections taken through the axes. A simple inspection of the figure will render this evident.* We first determine the side a opposite the angle A. Here (with A and B given, and a required), A becomes the *middle* part, and B and a the ***extremes disjunct***, or ***opposites***. Hence:

$R \cos A = \sin B \cos a$. And, consequently,

$$\cos a = \frac{R \cos A}{\sin B}.$$

* The less-experienced student is advised to fashion a solid triangle of this kind out of a piece of soft wood or chalk, and to mark upon its sides the outlines of the spherical triangles as given in the text: Figs. 13, 14, and 15.

Then to determine the side b (with A and B), B becomes the *middle* part, and A and b the *extremes disjunct* or *opposites*. Hence:

$$R \cos B = \sin A \cos b. \quad \text{And, consequently,}$$

$$\cos b = \frac{R \cos B}{\sin A.}.$$

With the two sides a and b, thus obtained, the axes x and $\breve{x}$ are readily calculated. As $\bar{x}$ = unity, $x = \cot a$, or $\log x = \log \cot a) - \log R$. An inspection of the figure will show this. Finally, axis $\breve{x} = (\tan b) \times x$; or $\log \breve{x} = (\log \tan b + \log x) - \log R$.

This understood, let us proceed, by way of example, to calculate the axial ratios of the three octahedrons in our crystal of sulphur, fig. 12.

1. *The Lower Form.*

In this form (lettered P), A (see figure 13) $= 53° \ 19'$; and $B = 42° \ 29'$. Then:

$$(\text{Log} \cos 53° \ 19') + 10 = 19.7762593$$
$$\text{Log} \sin 42° \ 29' \quad = \ \ 9.8295454$$

$$9.9467139 = \log \cos a = \log \cos 27°48'.$$
(seconds being neglected.

The log cot of this latter value $(27°48') = 10.2779915$. Deducting 10 (or log R) from this, and seeking for the corresponding number, we obtain, for axis x, the value 1.897.

Secondly:

$$(\text{Log} \cos 42°29') + 10 = 19.8677466$$
$$\text{Log} \sin 53°19 \quad = \ \ 9.9041470$$

$$9.9635996 = \log \cos b = \log \cos 23°8'.$$

The log tan of this angle $(23°8') = 9.6306556$. Adding the logarithmic value of x to this, and deducting 10 (or log R) from the sum, we obtain $(9.6306556 + 0.2779915) - 10 = \bar{1}.9086571 = \log. \breve{x} = \log. 0.8103$.

2. *The Middle Form.*

In this form, A becomes $63°30'$; and B, $56°35'$. Calculating from

these values, the axial ratio, as before,* we obtain for axis x, 0.6326; and for axis $\breve{x}$, 0.8102. Whilst, therefore, the latter axis remains of the same length as in the Lower Form, the vertical axis x, is only a third of the length of x in the form referred to. As the Lower Form is always present in crystals of Trimetric Sulphur, whilst the other forms are only occasionally present, we select it for our starting point or "protaxial form," and attach to it the simple symbol P. The symbol of the Middle Form then becomes $\frac{1}{3}$P. In the former, the axes x, $\bar{x}$, $\breve{x}$ are as 1.897 : 1 : 0.8103; in the latter, as 0.6326 : 1 : 0.8102.

3. *The Upper Form.*

In this form (see the angles at the commencement of § 8), $A = 71°2'$; and $B = 66°22'$.

$$(\text{Log cos } 71°2') + 10 = 19.5119074$$
$$\text{Log sin } 66°22' = 9.9619569$$
$$9.5499505 = \log \cos a = \text{(neglecting seconds) } \log \cos 69°13'.$$

$$\text{Log cot } 69°13' - 10 = \bar{1}.5792479 = \log x = \log 0.3795.$$

Secondly:

$$(\text{Log cos } 66°22') + 10 = 19.6030166$$
$$\text{Log sin } 71°2' = 9.9757570$$
$$9.6272596 = \log \cos b = \log \cos 64°55'9''.$$
$$\text{Log tan } 64°55'9'' = 10.3296891$$
$$\text{Log } x = \bar{1}.5792479$$
$$9.9089370$$
$$\text{Log } R = 10$$
$$\bar{1}.9089370 = \log \breve{x} = \log 0.8108.$$

The axes x, $\bar{x}$, $\breve{x}$, therefore, are to one another, as 0.3795 : 1 : 0.8108. Comparing these values with the axial ratios exhibited by the protaxial form P, the symbol of the present form becomes $\frac{1}{5}$ P.

* The formulæ, for more easy reference, are recapitulated here. Inclination over front edge = 2 A; over side edge = 2 B.

$$1\begin{cases}\text{Log cos } a = (\log \cos A, + 10) - \log \sin B;\\ \text{Log } x = \log \cot a, - 10.\end{cases}$$
$$2\begin{cases}\text{Log cos } b = (\log \cos B, + 10) - \log \sin A;\\ \text{Log } \breve{x} = (\log \tan b, + \log x) - 10.\end{cases}$$

9. The relations of the axes in these tri-polar forms or octahedrons, may also be calculated from the measured inclinations over a front and middle edge, or over a side and middle edge, as shown below. The student should work out these formulæ (from the angles given at the commencement of § 8) for the three sulphur forms: P, $\frac{1}{3}$P, $\frac{1}{5}$P.

1. Given the inclination over a front edge $= 2\,A$, and the inclination over a middle edge $= 2\,D$: required x and $\breve{x}$; $\bar{x}$ being unity.

To obtain $\breve{x}$, (see Fig. 14):

$$\text{Log cos } a = [(\log\cos A) + 10] - \log\sin D;$$
$$\text{Log } \breve{x} = (\log\cot a) - 10.$$

To obtain x, (see Fig. 14):

$$\text{Log cos } d = [(\log\cos D) + 10] - \log\sin A;$$
$$\text{Log } x = [(\log\tan.\ d) + \log\breve{x}] - 10.$$

2. Given the inclination over a side edge $= 2\,B$, and the inclination over a middle edge $= 2$ D: required x and $\breve{x}$; $\bar{x}$ being unity.

To obtain $\breve{x}$, (see Fig. 15);

$$\text{Log cos } b = [(\log\cos B) + 10] - \log\sin D;$$
$$\text{Log } \breve{x} = \log\tan b, -10.$$

To obtain x, (see Fig. 15):

$$\text{Log cos } d = [(\log\cos D + 10)] - \log\sin B;$$
$$\text{Log } x = (\log\tan d) - 10.$$

10. Figure 16 represents a crystal of Topaz, containing the following forms:

The Base, $B = x,\ \infty\,\bar{},\ \infty\,\breve{x}$.

Polar Forms.
- A polar or octahedron, $P = x,\ \bar{x},\ \breve{x}$.
- A second polar, $\frac{2}{3}P = \frac{2}{3}x,\ \bar{x},\ \breve{x}$.
- Another polar, belonging to the vertical zone of $V\,2 = \frac{3}{2}P2 = \frac{3}{2}x,\ \bar{x},\ 2\breve{x}$.
- A side-polar, or brachydome, $\breve{P} = x,\ \bar{x}\ \infty\ \breve{x}$.
- A second side-polar, $2\breve{P} = 2x,\ \bar{x},\ \infty\,\breve{x}$.

Vertical Forms:
- A vertical prism, $V = \infty\,x,\ \bar{x},\ \breve{x}$.
- A second vertical prism $V2 = \infty\,x,\ \bar{x},\ 2\breve{x}$.

Although these ~~prisms~~ forms may be recognised by the simple inspection of the crystal, as polars, side-polars, &c., their indices (or axial

ratios) as given in the figure, must be obtained by measurement and calculation. If, on account of its predominant occurrence, we select the front or inside vertical prism for the protaxial prism, and give it the simple symbol *V*, we know that the other prism will be *Vn* :—*n* referring to the comparative length of the shorter or frontal axis $\breve{x}$. In like manner, the polar or octahedron immediately above it, will be *mPn* ; with *n* alike in each form.* The actual value of *n*, however, and of *m*, must be sought for by calculation. In order to determine the axial ratios of the tri-polar forms after a principle differing from that adopted in §§ 8 and 9, we will base these calculations on the following measurements:

V : V (in front)	....	= 124°20′.
V2 : V2 (in front)	....	= 86°52.
B : $\breve{P}$†	....	= 154°13′.
B : 2$\breve{P}$	....	= 135°59′.
B : P	....	= 134°1′.
B : $\frac{2}{3}$P	....	= 145°24′.
B : $\frac{4}{3}$P2	....	= 135°4′.

Vertical Forms :

V : V = 124°20′. Half this, or 62°10′ equals *A* in figure 17. Then, as axis $\breve{x}$ equals unity—

Log $\breve{x}$ = (log cot 62°10′) − log $R = \bar{1}.7226207$ = log 0.5280.

V2 : V2 (in front) = 86° 52′. Therefore A' = 43°26′. Then—

Log $\breve{x}$ = (log cot 43°26′) − log R = 0.0237621 = log 1.056.

This value (1.056) being double that of axis $\breve{x}$ in V, selected for the protaxial prism, the symbol of the outer prism becomes V2.

Side-Polars or Brachydomes :

B on $\breve{P}$ = 154°13′. Deducting 90° from this, we obtain the angle *A* in figure 18. Then, $\breve{x}$ being unity—

Log x = (log cot 64°13′) − log $R = \bar{1}.6840011$ = log 0.4831.

* If, as in Fig. 7 (§ 6), this outer prism had been chosen for the protaxial one, the inside prism would have the notation V$\frac{1}{n}$; and the polars or octahedrons above it will be *m*P$\frac{1}{n}$. Where the indices *m*, *n* equal the axes x and $\breve{x}$, respectively, in the protaxial form, their value is understood to be 1, and, hence, is omitted in the notation.

† The side-polar or brachydome $\breve{P}$ in crystals of topaz, is a very rare form ; the two side-polars commonly present, being 2$\breve{P}$ and 4$\breve{P}$. These occupy, respectively, the places of $\breve{P}$ and 2$\breve{P}$ in the above figure. The inclination of B on 4$\breve{P}$ = 117°22′.

B on $2\breve{P}=135°59'$. This less $90°=A'$ in Fig. 18. Then—
Log $x'=$ (log cot $45°59'$)$-$log $R=\bar{1}.9850900=$ log 0.9662.

The upper form being assumed to be the protaxial one, this lower side-polar $=2\breve{P}$.

Polars or Octahedrons:

These, it will be remembered, are forms which cut the three axes. To determine the axial ratios of one of these octahedrons, without reference to any other form, we require at least two measurements, as shown in the example of the sulphur crystal, §§ 8 and 9. Having, however, in the present instance, the angle of the vertical prism V, we are able to determine the symbols of these octahedrons from their given angles with the basal plane B.

a).—B on P$=134°$ 1′. Deducting 90° from this, we obtain the angle A in figure 19. A' equals half the acute angle of V on V, or 90° minus half the front angle, the axes crossing at right angles. With this value, we first determine the length of a, a right line constituting the base of the triangle constructed from the given inclination of B on P. It cuts the edge between P and V at an angle of 90°, as shown in figures 19 and 20.

Here (as V : V$=124°$ 20′): $A'=90°-\frac{124°20'}{2}=27°\ 50'$. Then:

$$\text{Sin } 90° : \bar{x} :: \sin A' : a.$$

Consequently, as $\bar{x}=1$, and log sin $90°=10$:

$$\text{Log } a = (\log \sin 37°\ 50') - 10 = \bar{1}.6692250.$$

Secondly, in the determination of the vertical axis x,

R : cot A :: a : x. Therefore, as A$=134°$ 1′$-90°$,

$$\begin{array}{lr} \text{Log cot } 44°\ 1' \ \ldots\ldots\ldots\ldots & =10.0149100 \\ \text{Log } a \ \text{(as found above.)} & =\ \bar{1}.6692250 \\ \hline & 9.6841350. \\ \text{Log. } R \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots & =10 \\ \hline & \bar{1}.6841350 = \log.\ 0.4832, \end{array}$$

or the length of the vertical axis as determined from the side polar $\breve{P}$. The symbol of the present form, consequently, $=$P.

b.)—The inclination of the Base on the Upper Octahedron$=$ $145°24'$. In this form, consequently, $A=55°24'$; and $a=$ (as before) log $\bar{1}.6692250$.

$\therefore$ Log x=(log cot 55° 24′)+$\bar{1}$.6692250)−10=$\bar{1}$.5079821=log 0.3221.

This value is to that of the vertical axis in the protaxial form, as $\frac{1}{3}$ is to 1 : hence the symbol becomes $\frac{1}{3}$ P.

c.)—The remaining octahedron, it will be observed, lies above, and in the vertical zone of, the prism V 2. Hence, in this form, A′ (consult figures 19 and 20,) will equal $90° - \frac{V2 : V2}{2}$, or 46° 34′. Then to determine a:

Sin 90° : $\bar{x}$ :: sin 46° 34′ : a. Whence log a = log sin 46° 34′—10=$\bar{1}$.8610412.

Secondly, to determine the vertical axis x, we deduct 90°, as before, from the inclination of the Base on the form in question. This (see the measurements, as given above)=135° 4′. The angle A consequently (as in figure 19)=45° 4′. Then:

R: cot A :: a : x. Whence:

Log x = [(log cot A) + log a]—log R.

Log cot 45° 4′	=	9.9989893
Log a (as found above)...	=	$\bar{1}$.8610412
		9.8600305
Log. R	=	10.
		$\bar{1}$.8600305 = log 0.7245.

This is to the length of the vertical axis in the protaxial forms, as 1.5 : 1.0. Consequently, the symbol becomes $\frac{3}{2}$ P2.

The index 2, in this form, is known at once, by the form being in the vertical zone of the prism V2.

11. It frequently happens that, in a given crystal, only certain angles can be obtained by measurement: the other angles have then to be calculated. The following example will afford the student a general idea of the method of procedure in cases of this kind.

Given, in the crystal represented by figure 21, the inclination of V on V in front=102° 22′; and the inclination of P on V=126° 58′: required, the respective inclinations of P on P, over the front, side, and middle edges.

1). *To obtain the inclination of P : P over a ~~side~~ front polar edge.*

In the triangle constructed for this purpose, figure 22, $C = 90°$;
3rd line* inclination of V : V. A, on the other hand, = half the required
2nd line* D = the inclination of (P : V) − 90°; and a = half the front angle.

Then, to determine A from D and a; A becomes the *middle* part, and D and a the *extremes disjunct* or *opposites*. Whence:

$$R \cos A = \sin D \cos a. \quad \text{And, therefore,}$$

$$\cos A = \frac{\sin D \cos a.}{R}$$

$$
\begin{aligned}
\text{Log} \sin 36°58' &= 9.7791275 \\
\text{Log} \cos 51°11' &= 9.7971501 \\
\hline
&\ 19.5762776 \\
\text{Log } R \ \ldots\ldots\ldots &= 10 \\
\hline
\end{aligned}
$$

9.5762776 = Log. cos. $67°51'19''$. Twice this, or $135°42'38''$ equals the required inclination.

2). *To obtain the inclination of P : P over a side polar edge.*

In the constructed triangle, figure 23, C (as before) = 90°; D, again = the inclination of (P on V) −90°; and b = half the inclination of V : V over a side vertical edge, or $180° - \frac{103°28'}{2}$. B equals half the required angle.

With B, b, and D, B becomes the *middle* part, and b and D the *extremes disjunct* or *opposites*. Whence:

$$R \cos B = \sin D \cos b; \quad \text{and, consequently,}$$

$$\cos B = \frac{\sin D \cos b}{R}$$

$$
\begin{aligned}
\text{Log} \sin D = \text{log} \sin 36°58' &= 9.7791275 \\
\text{Log} \cos b = \text{log} \cos 38°49' &= 9.8916242 \\
\hline
&\ 19.6707517 \\
\text{Log } R \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots &= 10. \\
\hline
\end{aligned}
$$

9.6707517 = log cos $62°3'24''$.

Twice this, or $124°6'48''$ = the required inclination over the side polar edge.

8). *To obtain the inclination of P : P over a middle edge.*

Here no calculation is required. An inspection of the three preceding figures will show plainly that the angle in question $= D \times 2$; or, in other words, that the given angle (P : V) — 90° = one half the inclination of P : P over a middle edge. To render this still more clear, the vertical section, figure 24, is added. The required inclination, consequently = 73°56′

12. Finally, in bringing these examples to a close, we give, in the following formulæ, the means of calculating the more important angles of Trimetric forms, from the given ratios of the axes.

A (in figure 25) = half the angle over the front polar edge of an octahedron.

B = half the angle over a side polar edge.

D = half the angle over the middle edge; $D + 90° = m\,\mathrm{P} : \mathrm{V}$.

a = half the inclination of a front polar over the summit. $a + 90° = m\,\bar{\mathrm{P}}$: Base.

b = half this inclination over a middle edge. $b + 90° = m\,\bar{\mathrm{P}} : \bar{\mathrm{V}}$.

c and d =, respectively, the same angles in a side polar or brachydome. $c + 90° = m\,\breve{\mathrm{P}}$: Base; $d + 90° = m\,\breve{\mathrm{P}} : \breve{\mathrm{V}}$.

e = half the front angle of a vertical prism, V or V n; and f = the half side-angle of the same. $e + 90°$ = inclination of a V plane on $\bar{\mathrm{V}}$; $f + 90°$ = inclination of a V plane on $\breve{\mathrm{V}}$.

$$\text{Log tan } a = (\log \breve{x}) + 10 - \log x.$$
$$\text{Log cot } b = (\log \breve{x}) + 10 - \log x.$$
$$\text{Log cot } c = (\log x) + 10.$$
$$\text{Log tan } d = (\log x) + 10.$$
$$\text{Log cot } e = (\log \breve{x}) + 10.$$
$$\text{Log tan } f = (\log \breve{x}) + 10.$$
$$\text{Log cot } A = (\log \cot c + \log \sin a) - 10.$$
$$\text{Log cot } B = (\log \cot a + \log \sin c) - 10.$$
$$\text{Log tan } D = (\log \cot e + \log \sin f) - 10.$$

Example.—Required, in a crystal of Topaz, the inclination of the Base on the side polar $4\,\breve{\mathrm{P}}$.

In the protaxial form, $x = 0.4831$: see § 10, in which the axial ratios of Topaz are worked out. In $4\,\breve{\mathrm{P}}$, consequently, x must equal 1.9324.

To determine the inclination demanded, we require the angle c in figure 25.

$$\text{Log cot } c = (\log x) + 10 = \log \cot 27^\circ 22'.$$

This $+ 90^\circ$ (see the explanation of fig. 25, above) $= 117^\circ 22'$, as already given in a foot-note on a preceding page.

13. The equations for the determination of the octahedral half-angles A, B, D, (see § 12, above), may be also expressed as follows:

$$\text{Cos } A = \frac{\breve{x}\ x}{\sqrt{(\breve{x}^2 + x^2 + \breve{x}^2 x^2)}}$$

$$\text{Cos } B = \frac{x}{\sqrt{(\breve{x}^2 + x^2 + \breve{x}^2 x^2)}}$$

$$\text{Cos } D = \frac{\breve{x}}{\sqrt{(\breve{x}^2 + x^2 + \breve{x}^2 x^2)}}$$

ADDITIONAL REMARKS.

1. Some authors, in calculating the axial ratios of crystal-forms, make the vertical axis equal to unity; whilst others, in Trimetric forms, make the shorter horizontal axis, and in Monoclinic forms, the inclined axis, $= 1.0$. The processes, however, by which the axial ratios are obtained, are in all cases the same: a slight alteration or transposition in the formulæ being alone required in changing from one unity-system to the other. Thus, in a trimetric prism, if $\breve{x}$ be assumed to equal 1, and $a =$ half the front or obtuse angle of V : V; axis $\bar{x} = \tan a$. See § 6 above.

2. In calculating the axial ratios of forms belonging to the Monoclinic System, the obliquity of the back-and-front axis has to be taken into consideration. The acute angle made by this axis on the vertical axis, always forms an element in the computations. The angle in question equals the inclination of B on $\overline{\text{V}}$, behind. If it be denoted by k; the vertical axis by x; the ortho-axis by $\bar{x}$; the clino-axis by $\ddot{x}$; and half the prism-angle V : V in front, by a; then (with $\bar{x}$ unity), $\log \ddot{x} = (\log \cot a) - \log \sin k$. Or (with $\ddot{x}$ unity) $\log \bar{x} = (\log \sin k) - \log \cot a$.

3. Figures 26 to 29 illustrate two different methods (first published by the writer) of determining the vertical axis in Rhombohedrons, from the measured angle over a polar edge (= R : R.)

1) In figure 26, $A = 60°$ (or an angle equal to that between two of the horizontal axes); $B = \frac{R:R}{2}$ (or half the inclination over a polar edge); and $C = 90°$. The side b is required:

$$\cos b = \frac{\cos B}{\sin A}.$$

In figure 27, $A = 30°$ (or half the space between two horizontal axes); $b = b$ as found by the first equation; and $C = 90°$. The part c is required:

$$\cot c = \frac{\cos A}{\tan b}.$$

The cotangent thus obtained, as will be seen by an inspection of figure, gives the value of axis x: the horizontal axes being unity.

These formulæ may be transformed, for their more ready working, into the following:

$$\log \cos b = (\log \cos B) + 0.0624694;$$
$$\log x = (\log \cot b) - 10.0624694.$$

2) In figure 28, which corresponds with figure 30, $A = 60$; $B = \frac{R:R}{2}$; and $C = 90°$. c, or the inclination of a polar edge on the vertical axis, is the part required:

$$\cos c = \cot A \cot B.$$

In figure 23, $A = 90°$ $(= 60° + 30°)$; $B = \frac{R:R}{2}$; and $c =$ the inclination of a polar edge on the vertical axis, as found by the preceding operation. b is the part required:

$$\tan b = \frac{\sin c}{\cot B}. \quad \text{Finally, } x = \cot b.$$

These formulæ, as in case 1, may be expressed logarithmically, as follows:

$$\log \cos c = (\log \cot B) - 0.2385606;$$
$$\log x = (\log \cot c) - 9.7614394.$$

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Fig. 1.

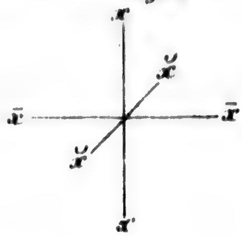


Fig. 2.

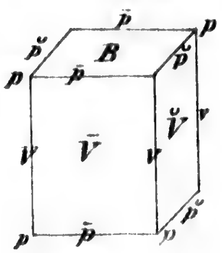
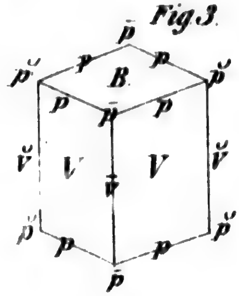


Fig. 3.



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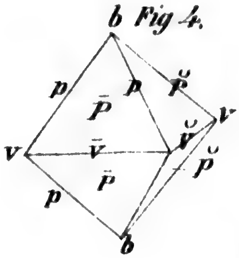

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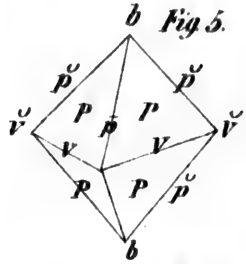

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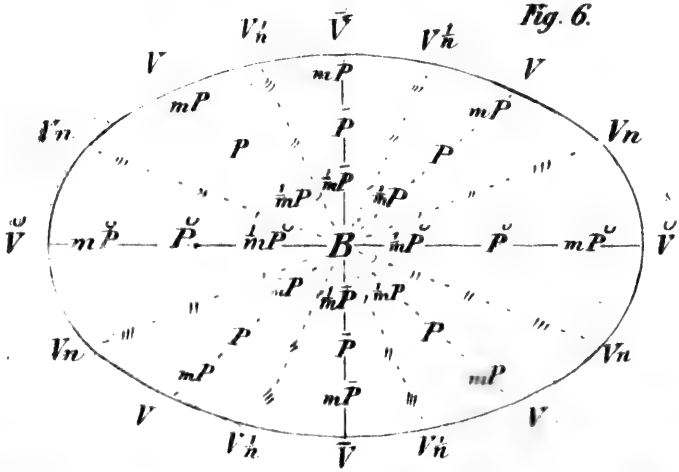

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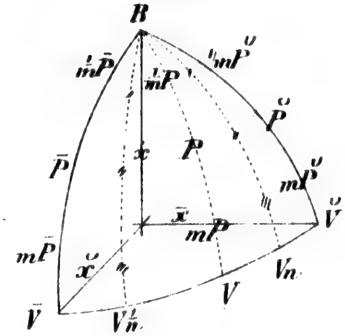
Fig 6.*



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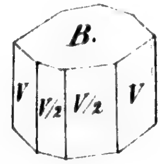

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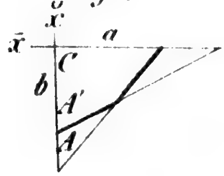

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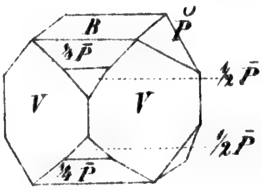

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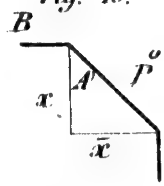

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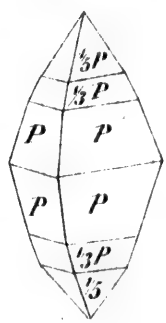

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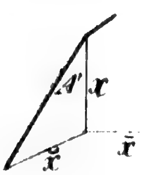

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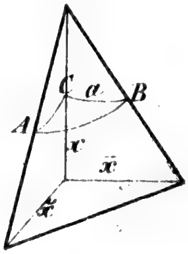
Fig. 13.



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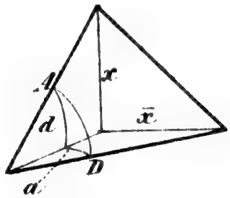

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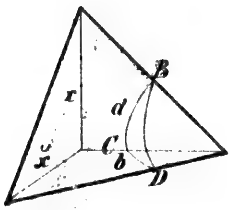

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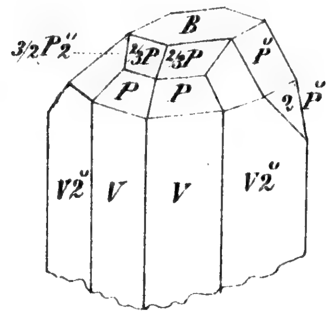

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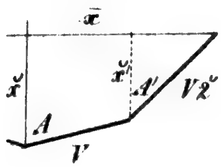

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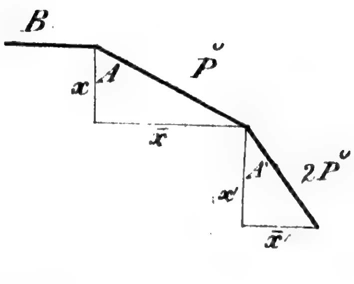

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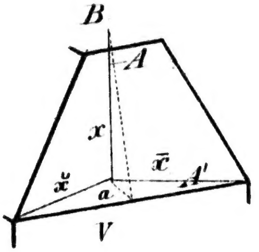
Fig. 19.



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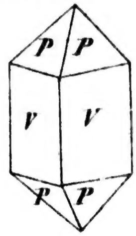

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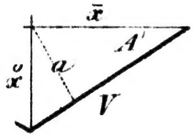

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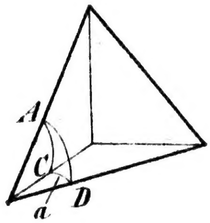
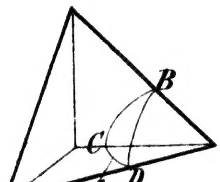


Fig 23



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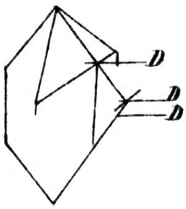

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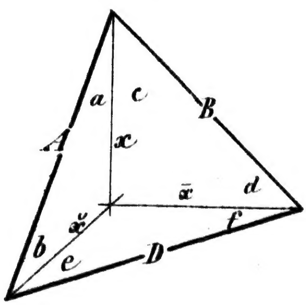

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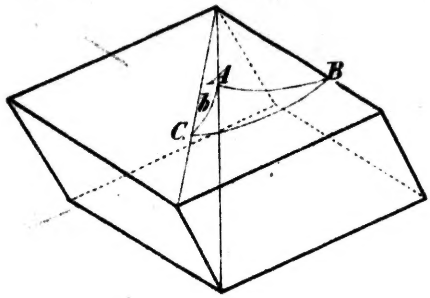
Fig. 26.



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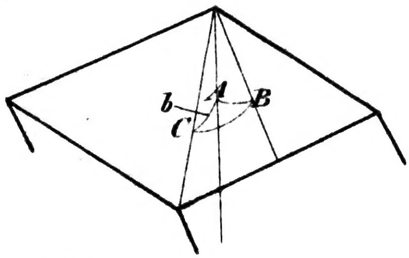

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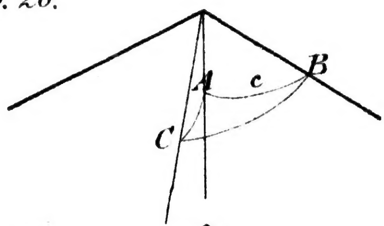

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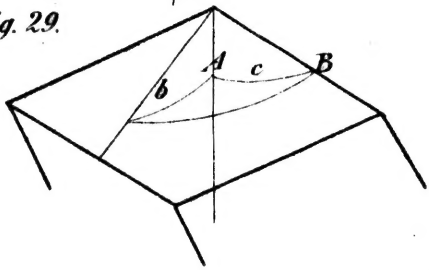

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